Towards Backward Fuzzy Rule Interpolation

Jin, Shangzhu; Diao, Ren; Shen, Qiang

Published in:
Proceedings of the 11th UK Workshop on Computational Intelligence

Publication date:
2011

Citation for published version (APA):
Jin, S., Diao, R., & Shen, Q. (2011). Towards Backward Fuzzy Rule Interpolation. In *Proceedings of the 11th UK Workshop on Computational Intelligence* (pp. 194-200). Manchester University Press.
http://hdl.handle.net/2160/7591

Towards Backward Fuzzy Rule Interpolation

Shangzhu Jin, Ren Diao and Qiang Shen

Abstract— **Fuzzy rule interpolation (*FRI*) is well known for reducing the complexity of fuzzy models and making inference possible in sparse rule-based systems. However, in practical fuzzy applications with inter-connected rule bases, situations may arise when a crucial antecedent of observation is absent, either due to human error or difficulty in obtaining data, while the associated conclusion may be derived according to different rules or even observed directly. To address such issues, a concept termed Backward Fuzzy Rule Interpolation (*B-FRI*) is proposed, allowing the observations which directly relate to the conclusion be inferred or interpolated from the known antecedents and conclusion. B-FRI offers a way to broaden the fields of research and application of fuzzy rule interpolation and fuzzy inference. The steps of *B-FRI* implemented using the scale and move transformation-based fuzzy interpolation are given, along with two numerical examples to demonstrate the correctness and accuracy of the approach. Finally, a practical example is presented to show the applicability and potential of *B-FRI*.**

I. Introduction

The Compositional Rule of Inference (*CRI*) [17] plays a predominate role in fuzzy systems, where fuzzy rules are typically interpreted as fuzzy relations. Many different *CRI* implementations have been proposed by employing different t-norms and s-norms [8]. However, all such implementations are only applicable for problem domains where significantly dense rule bases are available. Fuzzy rule interpolation (*FRI*) has been introduced to address this limitation [10], and is well known for reasoning in the presence of insufficient knowledge commonly referred to as sparse rule bases. Various interpolation methods have been developed in the literature [7], [14], most of which can be categorised into two classes with several exceptions (e.g. type II fuzzy interpolation [4]).

The first category of approaches directly interpolates rules whose antecedent is identical to the given observation. The consequence of the interpolated rule is thus the logical outcome. Most typical approaches in this group [2], [9], [10] are based on the Decomposition Principle and Resolution Principle, which assumes that a fuzzy set can be represented by a series of α-cuts ($\alpha \in (0,1]$). The α-cut of the interpolated consequent fuzzy set is then calculated from the α-cuts of the observed antecedent fuzzy sets, and all the fuzzy sets involved in the rules used for interpolation. Having found the consequent α-cuts for all $\alpha \in (0,1]$, the consequent fuzzy set can be easily assembled by applying the Resolution Principle.

The second category is based on the analogical reasoning mechanism [3], usually referred to as "analogy-based fuzzy interpolation". These approaches [1], [5], [6] first interpolate an artificially created intermediate rule, such that the antecedent of the intermediate rule is "closer" to the given observation. Then, a conclusion can be deduced by firing this intermediate rule through the analogical reasoning mechanism. The shape distinguishability between the resulting fuzzy set and the consequence of the intermediate rule, is then analogous to the shape distinguishability between the observation and the antecedent of the created intermediate rule.

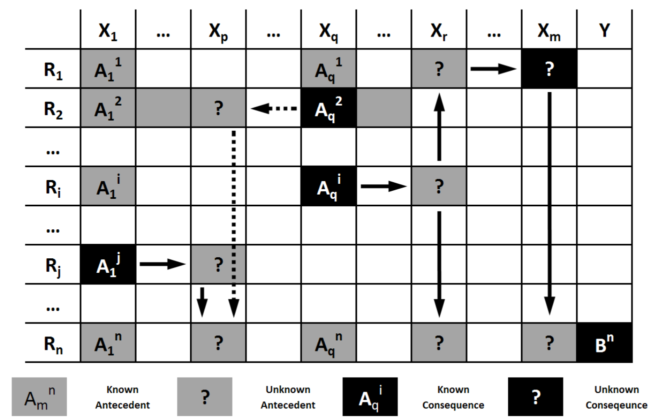

Fig. 1. A general backward fuzzy rule interpolation structure

Despite the numerous approaches present and their advantages, *FRI* techniques are relatively rarely applied in practice [12]. One of the main reasons is that many practical fuzzy applications are multiple-input and multiple-output (*MIMO*) systems. The rule bases involved may be irregular in nature, and could be arranged in an inter-connected mesh, where observations and conclusions in between different rule bases could be overlapped, and yet not directly relative. For such complex systems, any missing values in a given set of observations could cause a complete failure in interpolation. For instance, in Fig. 1, the conclusion B^n of the final rule R_n can not be interpolated straightforwardly, because the three missing observations cannot be deduced using conventional means.

To dress this kind of problem, a novel concept termed Backward Fuzzy Rule Interpolation (*B-FRI*) is proposed. *B-FRI* argues that the unknown antecedents of observation can be interpolated, provided that all other antecedents and the conclusion are known. Being a beneficial addition to *FRI*, *B-FRI* is an approach which achieves indirect interpolative reasoning. Using the earlier example in Fig. 1, the unknown antecedents A_r^n and A_p^n can be backward interpolated according to rules R_j and R_i, where the conclusions A_1^j, A_q^i

Shangzhu Jin, Ren Diao and Qiang Shen are with the Department of Computer Science, Aberystwyth University, UK (email: {shj1, rrd09, qqs}@aber.ac.uk).

and other antecedent terms are already known. The last missing antecedent A_m^n can then be interpolated using R_1, and subsequently B^n can also be deduced, as now all required antecedents are known for forward interpolation.

In this paper, *B-FRI* is implemented using the scale and move transformation-based fuzzy interpolative reasoning (*T-FIR*) [5], [6], which is an analogy-based approach. The main reason for this adoption is that *T-FIR* offers a flexible and complete means to handle both interpolation and extrapolation involving multiple fuzzy rules. *T-FIR* guarantees the uniqueness as well as normality and convexity of the resulting interpolated fuzzy sets. It is also able to handle interpolation of multiple antecedent variables with different types of fuzzy membership function.

The rest of this paper is organised as follows. Section II reviews the general concepts of the *T-FIR*. The proposed *B-FRI* approach is given in Section III, including methods for single and multiple antecedent variables, along with worked examples. A possible application that reasons about terrorist activities is provided in Section IV, to demonstrate the correctness and accuracy of this approach. Section V concludes the paper and suggests possible future enhancements.

II. Background of Transformation-Based Interpolative Reasoning

This section provides a general introduction of the procedures involved in *T-FIR* [6], including the definition of the underlying key concepts, and an outline of its interpolation steps. Triangular membership functions are the most common and widely used fuzzy set representation in fuzzy systems, and they are also adopted in this paper for simplicity.

The key concept used in *T-FIR* is the representative value $Rep(A)$ of a triangular fuzzy set A. It is defined as the average of the X coordinates of the triangle's three points: the left and right extreme points a_0, a_2 (with membership values $= 0$), and the normal point a_1 (with membership value $= 1$).

$$Rep(A) = \frac{a_0 + a_1 + a_2}{3} \tag{1}$$

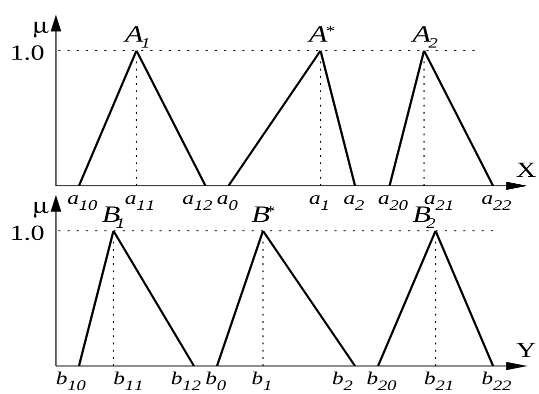

Fig. 2. Interpolation with triangular membership functions.

A. T-FIR with Two Single Antecedent Rules

1) *Determination of Two Closest Rules*
For single antecedent rules $A \Rightarrow B$, the distances to the observation A^* can be computed using Eqn. 2.

$$d = d(A, A^*) = d(Rep(A), Rep(A^*)) \tag{2}$$

2) *Construct Intermediate Fuzzy Terms*
Suppose that the two neighbouring rules after distance comparison $A_1 \Rightarrow B_1$, $A_2 \Rightarrow B_2$, and the observation A^* are given as illustrated in Fig. 2. The intermediate fuzzy term $A' = (1-\lambda_A)A_1 + \lambda_A A_2$ can then be defined according to the ratio of distances λ_A between their representative values, and $Rep(A') = Rep(A^*)$:

$$\lambda_A = \frac{d(A_1, A^*)}{d(A_1, A_2)} = \frac{d(Rep(A_1), Rep(A^*))}{d(Rep(A_1), Rep(A_2))} \tag{3}$$

$$\begin{cases} a_0' = (1-\lambda_A)a_{10} + \lambda_A a_{20} \\ a_1' = (1-\lambda_A)a_{11} + \lambda_A a_{21} \\ a_2' = (1-\lambda_A)a_{12} + \lambda_A a_{22} \end{cases} \tag{4}$$

Similarly the fuzzy set B' on the consequence domain can be obtained. In the single antecedent case, $\lambda_B = \lambda_A$.

$$B' = (1-\lambda_B)B_1 + \lambda_B B_2 \tag{5}$$

3) *Scale Transformation:*
Let $A'' = (a_0'', a_1'', a_2'')$ denote the fuzzy set generated by the scale transformation. By using the scale rate s_A, A''s current support (a_0', a_2') is transformed into a new support (a_0'', a_2''), such that $a_2'' - a_0'' = s_A \times (a_2' - a_0')$.

$$\begin{cases} a_0'' = \frac{a_0'(1+2s_A) + a_1'(1-s_A) + a_2'(1-s_A)}{3} \\ a_1'' = \frac{a_0'(1-s_A) + a_1'(1+2s_A) + a_2'(1-s_A)}{3} \\ a_2'' = \frac{a_0'(1-s_A) + a_1'(1-s_A) + a_2'(1+2s_A)}{3} \\ s_A = \frac{a_2'' - a_0''}{a_2' - a_0'} \end{cases} \tag{6}$$

4) *Move Transformation:*
The current support of A'' is moved to (a_0, a_2) while keeping its representative value, resulting in the fuzzy set A^*.

$$\begin{cases} m_A = \frac{a_0 - a_0''}{\frac{a_1'' - a_0''}{3}}, a_0 \geq a_0'' \\ m_A = \frac{a_0 - a_0''}{\frac{a_2'' - a_1''}{3}}, otherwise \end{cases} \tag{7}$$

Given the move ratio m_A, the transformed fuzzy set A^* can be calculated using:

$$\begin{cases} \begin{cases} a_0 = a_0'' + m_A \frac{a_1'' - a_0''}{3} \\ a_1 = a_1'' - 2m_A \frac{a_1'' - a_0''}{3} \\ a_2 = a_2'' + m_A \frac{a_1' - a_0''}{3} \end{cases} & m_A \geq 0 \quad \text{(8a)} \\ \begin{cases} a_0 = a_0'' + m_A \frac{a_2'' - a_1''}{3} \\ a_1 = a_1'' - 2m_A \frac{a_2'' - a_1''}{3} \\ a_2 = a_2'' + m_A \frac{a_2'' - a_1''}{3} \end{cases} & otherwise \quad \text{(8b)} \end{cases}$$

5) The above transformations from $A^{'}$ to A^* can be concisely represented by function $T(A^{'},A^*)$. Similarly, the function T is applied to transforming $B^{'}$ to B^* such that:

$$T(B^{'},B^*) = T(A^{'},A^*) \tag{9}$$

where $s_B = s_A$ and $m_B = m_A$ for the current single antecedent case.

B. T-FIR with Multiple Antecedent Variables

1) *Determination of Two Closest Rules*
Without losing generality, rules R_i, R_j and observation O can be represented in the following forms:

R_i: IF x_1 is A_1^i, $\cdots$, x_k is A_k^i, $\cdots$, x_M is A_M^i, THEN y is B^i
R_j: IF x_1 is A_1^j, $\cdots$, x_k is A_k^j, $\cdots$, x_M is A_M^j, THEN y is B^j
O: A_1^*, $\cdots$, A_l^*, $\cdots$, A_M^*

where A_k^i is the linguistic term of the R_i rule on the k^{th} antecedent dimension, $k = 1,\cdots,M$. A_l^*, $l = 1,\cdots,M$ are the observed fuzzy sets of variable x_l. and M is the total number of antecedents. The distance d_k between the fuzzy sets A_k^i and A_k^* can then be calculated as:

$$d_{A_k} = \frac{d(A_k^i,A_k^*)}{max_{A_k} - min_{A_k}} = \frac{d(Rep(A_k^i),Rep(A_k^*))}{max_{A_k} - min_{A_k}} \tag{10}$$

where max_{A_k} and min_{A_k} are the maximal and minimal domain values of variable x_k. This normalises the absolute distance measure into the range [0,1], so that distances are compatible with others measured over different domains. From this, the distance d between a rule and an observation can then be calculated as the average of all variables' distances. The two rules which have the minimum distance are chosen, which are located on both sides of the observation, respectively.

$$d = \sqrt{{d_{A_1}}^2 + {d_{A_2}}^2 + \cdots + {d_{A_M}}^2} \tag{11}$$

2) *Interpolation between the Two Rules*
Suppose that the two adjacent rules are R_i and R_j, to interpolate B^*, the values A_k^i and A_k^j are used in Eqn. 3 and 4 to obtain the displacement factor λ_{A_k}, and the intermediate fuzzy terms $A_k^{'}$ for each antecedent dimension x_k. In conjunction with the given observation terms A_k^*, the scale and move transformation $T(A_k^{'},A_k^*)$, and the necessary parameters involved s_{A_k} and m_{A_k} are calculated using Eqn. 6 and 7.
For the current scenario with multiple antecedent rules, each antecedent dimension would have its own λ_{A_k}, s_{A_k}, and m_{A_k} values. The following equations aggregate them all in order to discover the intermediate fuzzy term $B^{'}$. The fuzzy set B^* of conclusion can then be estimated by the transformation $T(B^{'},B^*) = \{s_B, m_B\}$.

$$\lambda_B = \frac{1}{M}\sum_{k=1}^{M} \lambda_{A_k} \tag{12}$$

$$B^{'} = (1-\lambda_B)B^i + \lambda_B B^j \tag{13}$$

$$s_B = \frac{1}{M}\sum_{k=1}^{M} s_{A_k} \tag{14}$$

$$m_B = \frac{1}{M}\sum_{k=1}^{M} m_{A_k} \tag{15}$$

III. Backward Fuzzy Rule Interpolation

In this section, the concept of Backward Fuzzy Rule Interpolation (*B-FRI*) is presented. Again, for simplicity, only cases involving two adjacent rules are considered. In general, any *FRI* can be represented as follows:

$$B^* = f_{FRI}(A_1^*,\cdots,A_l^*,\cdots,A_M^*,(R_i,R_j)) \tag{16}$$

where f_{FRI} denotes the entire process of the forward fuzzy rule interpolation using rules R_i and R_j, A_l^*, $l = 1,2,\cdots,M$ are observed values of the antecedent variables and B^* is the interpolated conclusion, (R_i,R_j) are the two adjacent rules. Similarly, for *B-FRI* the following general form can be used:

$$A_l^* = f_{B-FRI}((B^*,(A_1^*,\cdots,A_{l-1}^*,A_{l+1}^*,\cdots,A_M^*)),(R_i,R_j)) \tag{17}$$

where f_{B-FRI} denotes the entire process of backward fuzzy interpolation, and A_l^* is the unknown observation to be backward interpolated.

A. B-FRI with Single Antecedent Variable

By definition of linear interpolation, the process of *B-FRI* with single antecedent rules is identical to that of traditional interpolation. The antecedents and consequence of rules are simply positioned in the reverse order, and the observed consequent variable becomes the new antecedent.

Example 3.1: An example is used here to illustrate the process, as well as to provide an example of the *T-FIR* procedures. The two original rules are given in Table I and illustrated in Fig. 3, with the conclusion being given: $B^* = (5.83, 6.26, 7.38)$. Note that in contrary to the notations given in Section II-A, the conclusion B^* is now known, and A^* becomes the targeted interpolation result.

TABLE I
Example of *B-FRI*, $B^* = (5.83, 6.26, 7.38)$

Rule	Antecedents	Consequence
R_1	$A^1 = (0,5,6)$	$B^1 = (0,2,4)$
R_2	$A^2 = (11,13,14)$	$B^2 = (10,11,13)$

1) *Construct Intermediate Fuzzy Terms $B^{'}$ and $A^{'}$:* The relative placement factor $\lambda_B = 0.481$ is calculated first using Eqn. 3. The intermediate fuzzy term $B^{'} = (4.811, 6.330, 8.330)$ is constructed according to Eqn. 4, which has the same representative value as

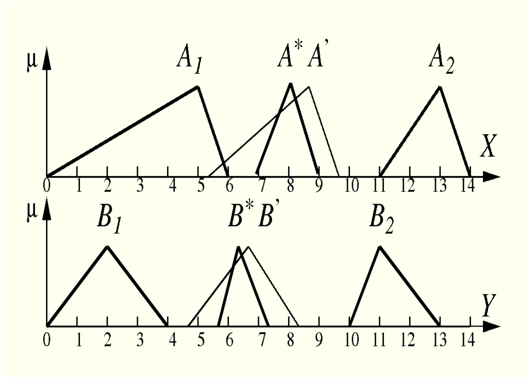

Fig. 3. Example of *B-FRI* with single antecedent

the conclusion B^*, $Rep(B^{'}) = Rep(B^*) = 6.490$. According to Eqn. 5, work out the intermediate value $A^{'} = (5.292, 8.849, 9.849)$.

2) *Calculate Scale Rate:* The scale rate s_B is calculated using Eqn. 6, resulting in $s_B = 0.440$. The second intermediate term $B^{''} = (5.75, 6.42, 7.30)$ denote the fuzzy set generated by the scale transformation. This transformation rescales the support of $B^{'}$, $(b_0^{'}, b_2^{'}) = (4.81, 8.33)$ into a new support $b_0^{''}, b_2^{''} = (5.75, 7.30)$, such that the length of support is modified by s_B: $(7.30 - 5.75) = 1.55 = 0.440 \times (8.33 - 4.81)$.
3) *Calculate Move Ratio:* According to Eqn. 7, $m_B = 0.357$ can be deduced that will shift $(b_0^{''}, b_2^{''}) = (5.75, 7.30)$ to $(b_0^*, b_2^*) = (5.83, 7.38)$. The result of the above scale and move transformation should successfully transform $B^{''}$ back to B^*.
4) *Scale and Move Transformation $A^{'}$ to A^*:* Having discovered s_B and m_B, the reverse transformation of Eqn. 9 can be performed. The scale transformation is first applied to $A^{'}$ using $s_A = s_B = 0.440$, resulting in the second intermediate term $A^{''} = (6.805, 8.372, 8.812)$. In the end, $A^* = (6.992, 7.999, 8.999)$ is a result from the move transformation from $A^{''}$ using move ratio $m_A = m_B = 0.357$, hereby completing the backward transformation process $T(A^{'}, A^*) = T(B^{'}, B^*)$.

The correctness and accuracy of backward interpolation can be easily proven by doing conventional *T-FIR* using $A^* = (6.992, 7.999, 8.999)$ as the observed value. The conclusion $B^* = (5.8304, 6.2604, 7.3803)$ consistent with the originally given observation.

B. B-FRI with Multiple Antecedent Variables

A close examination of the *T-FIR* algorithm reveals that all the parameters in association with the calculation of the consequence variable, namely λ_B, s_B and m_B, are algebraic averages of the parameters from individual antecedent terms according to Eqn. 12, 14 and 15. Thus, it has an intuitive appeal to assume that, in order to perform backward interpolation, the consequent variable should be treated with a biased weight that is the sum of all antecedent weights. The parameters for the missing antecedent should then be calculated by subtracting parameter values of the known antecedents from the consequent values. The following summarises the proposed *B-FRI* algorithm that reflects this intuition:

1) *Determination of Closest Rules* In reference to the earlier definition of the *B-FRI* process in Eqn. 17, when $B^*, (A_1^*, \cdots, A_{l-1}^*, A_{l+1}^*, \cdots, A_M^*)$ are given, in order to interpolate the unknown antecedent A_l^*, the discovery of the closest rules R_i and R_j are required. In contrary to the plain distance measure introduced in Eqn. 10 and 11, a modified scheme is proposed which reflects the biased consideration toward the consequent variable:

$$d = \sqrt{\left(\sum_{k=1}^{M} w_{A_k}\right) \times {d_B}^2 + \sum_{k=1, k\neq l}^{M} \left(w_{A_k} {d_{A_k}}^2\right)} \tag{18}$$

When used purely for the choice of closest rules, the square root stated in the original distance measure becomes unnecessary, as only the ordering information is recognised. Further more, for general backward interpolation without sufficient knowledge on the actual level of importance of different antecedents, all antecedents should be treated equal, that is:

$$w_{A_1} = w_{A_2} = \cdots = w_{A_M} = 1 \tag{19}$$

$$w_B = \sum_{k=1}^{M} w_{A_k} = M \tag{20}$$

The above formula can therefore be simplified into:

$$\hat{d} = M \times {d_B}^2 + \sum_{k=1, k\neq l}^{M} {d_{A_k}}^2 \tag{21}$$

For better illustration of the later interpolation procedures, the two adjacent rules R_i and R_j, and the observation O are represented as follows:

R_i: IF x_1 is A_1^i, $\cdots$, x_k is A_k^i, $\cdots$, x_M is A_M^i,
THEN y is B^i
R_j: IF x_1 is A_1^j, $\cdots$, x_k is A_k^j, $\cdots$, x_M is A_M^j,
THEN y is B^j
O: A_1^*, $\cdots$, A_{l-1}^*, A_{l+1}^*, $\cdots$, A_M^*, B^*

where A_k^i and A_k^j denote the k^{th} terms of rules R_i and R_j respectively. The missing antecedent value in the observation is denoted by A_l^*.

2) *Construct the Intermediate Fuzzy Terms* The first step of the actual interpolation process is to compute the intermediate fuzzy terms for each antecedent and the consequent variable, including $A_l^{'}$ for the missing l^{th} antecedent. For this, λ_{A_l} is needed to be calculated. Recall Eqn. 12 where λ_l was part of the $\sum_{k=1}^{M} \lambda_{A_k}$ on the right hand side. Now that B^* is already known, λ_B can be easily calculated from Eqn. 22. It is then possible to deduce λ_{A_l} according to Eqn. 23, where each λ_{A_k}

for the other known antecedents A_k is calculated using the generalised version of Eqn. 3, shown in Eqn. 24. The intermediate fuzzy terms $A_k^{'}$, $k = 1, 2, \cdots, M$ and $B^{'}$ can now be computed according to Eqn. 4.

$$\lambda_B = \frac{d(Rep(B^i), Rep(B^*))}{d(Rep(B^i), Rep(B^j))} \tag{22}$$

$$\lambda_{A_l} = M \times \lambda_B - \sum_{k=1,k\neq l}^{M} \lambda_{A_k} \tag{23}$$

$$\lambda_{A_k} = \frac{d(A_k^i, A_k^*)}{d(A_k^i, A_k^j)} = \frac{d(Rep(A_k^i), Rep(A_k^*))}{d(Rep(A_k^i), Rep(A_k^j))} \tag{24}$$

3) *Scale and Move Transformation* Having the intermediate fuzzy terms, the essential parameters s_{A_l} and m_{A_l} involved in the transformation process can be derived. Following the same reasoning and steps as that of λ_{A_l}, by reversing the forward transformation procedure introduced in Eqn. 14 and 15, the required values can be found as shown below according to Eqn. 25 and 26, where s_B and m_B are immediately obtainable by evoking Eqn. 6 and 7. The generalised formulae for s_{A_k} and m_{A_k} can be derived similarly to Eqn. 24, such that:

$$s_{A_l} = M \times s_B - \sum_{k=1,k\neq l}^{M} s_{A_k} \tag{25}$$

$$m_{A_l} = M \times m_B - \sum_{k=1,k\neq l}^{M} m_{A_k} \tag{26}$$

4) Finally with all parameters acquired, the scale and move transformation on $A_l^{'}$ can be performed as shown in Eqn. 27, resulting in the (backward) interpolated value A_l^*, that was originally missing.

$$T(A_l^{'}, A_l^*) = \{s_{A_l}, m_{A_l}\} \tag{27}$$

Example 3.2: This example illustrates backward interpolation of multiple antecedent variables with triangular membership functions. The two adjacent rules are given in Table II and Fig. 4, with the observation being

$$A_1^* = (4,5,6), A_2^* = (5,6,7), B^* = (10.23, 11.80, 13.73)$$

TABLE II

TWO CLOSEST RULES FOR OBSERVATION

Rule	Antecedents	Consequence
Rule 1	$A_1^1 = (1,2,3)$, $A_2^1 = (2,3,4)$, $A_3^1 = (2,4,5)$	$B^1 = (5,7,9)$
Rule 2	$A_1^2 = (7,9,10)$, $A_2^2 = (8,9,10)$, $A_3^2 = (9,10,11)$	$B^2 = (15,17,19)$

1) *Construct Intermediate Fuzzy Terms:*
The λ_B for the consequent dimension is 0.492 according to Eqn. 22, the parameter for the missing observation can then be calculated using Eqn. 23. $\lambda_{A_3} = M \times \lambda_B - \sum_{k=1,k\neq l}^{M} \lambda_{A_k} = 3 \times 0.492 - (0.450 + 0.5) = 0.526$, and the intermediate fuzzy set $A_3^{'} = (5.684, 7.158, 8.158)$ can be obtained according to Eqn. 4.

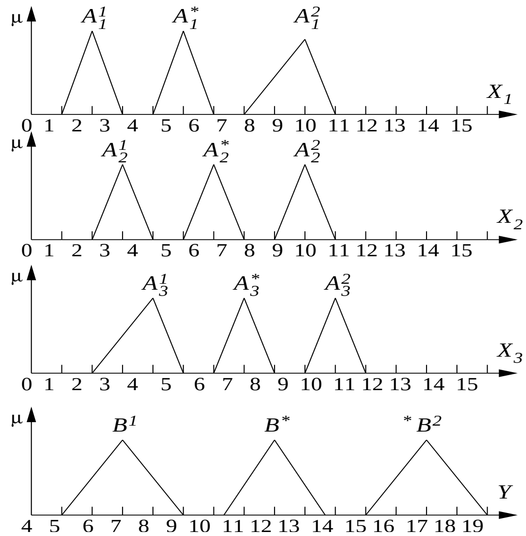

Fig. 4. Example of *B-FRI* with multiple antecedents

2) *Scale and Move $A_3^{'}$ to A_3^*:*
The individual scale and move parameters can be calculated according to Eqn. 6 and 7, resulting in $s_B = 0.875$, $m_B = 0.103$. From Eqn. 25 and 26 it is possible to obtain $s_{A_3} = 0.809$ and $m_{A_3} = 0.153$. The second intermediate fuzzy term $A_3^{''}$ can be computed to be $(5.934, 7.126, 7.934)$. Finally, according to Eqn. 27, the transformed $A_3^* = (5.995, 7.004, 7.995)$ can be obtained.

Again the result can be validated by performing the conventional *T-FIR* using the obtained A_3^*, resulting in the conclusion being $(10.2301, 11.8001, 13.7298)$, which is consistent with the given observed conclusion.

IV. Experimentation and Discussion

A. Experimental Scenario

Consider a practical scenario that involves the possible detection of terrorist bombing events. The likelihood of an explosion can be directly affected by the number of people in the area, a crowded place is usually more likely to attract terrorists' attention. The number of public warning signs displayed in the area may also affect the potential outcome. With many eye-catching warning signs, people may raise their awareness of the surroundings, and may promptly report suspicious individuals or items, giving less opportunities for the terrorists to attack. Given such concepts regarding the antecedent variables **Crowdedness**, **Warning** level, and the consequent variable **Explosion Likelihood**, a rule base can be established with example rules listed in Table III.

Additionally, there may exist another rule base that focuses on the prediction of crowdedness. The number of people in an area is directly related to the **Popularity** of the place, but

	Crowdedness	**Warning**	**Explosion Likelihood**
C1	Very Low	Moderate	Very Low
C2	Moderate	Moderate	Low
C3	Moderate	High	Low
C4	High	Moderate	Moderate

TABLE III

EXAMPLE RULES FOR EXPLOSION LIKELIHOOD

	Popularity	**Travel**	**Warning**	**Crowdedness**
C1	Very Low	Very Low	Very High	Very Low
C2	Very Low	Very High	Very Low	Low
C3	Very Low	High	High	Low
C4	Moderate	Moderate	High	Low
C5	Moderate	High	Low	Moderate
C6	High	Very Low	High	Very Low
C7	High	High	Moderate	Moderate
C8	High	High	Very Low	High

TABLE IV

EXAMPLE RULES FOR CROWDEDNESS

it can also be affected by the level of **Travel** Convenience. A well known place that is hard to get to by usual means, may not have more people than an attraction that is less famous, but very easy to reach. Moreover, considering the current scenario, the crowdedness may also change in relation to the **Warning** level. The less brave individuals may shy away from places that are considered dangerous, judged by the amount of explosion alerts in the surrounding areas. Summarising the above information, the second rule base may be derived. Table IV displays a subset of such rules.

It is easy to identify that both rule bases involved are fuzzy and sparse. Fuzziness is naturally introduced by the presence of linguistic terms used to describe the domain variables. From the linguistic terms it can be deduced that the rule bases contain substantial gaps amongst the underlying domains of the variables concerned. The terms *Low*, *Moderate*, and *High* although provide reasonable coverage of the entire domain, but intermediate values such as *Moderate Low*, *Moderate High* are not represented. By converting the linguistic terms into fuzzy sets, the following two rule bases presented in Table V and VI may be produced. These are used for later experiments for illustrative purposes.

For traditional forward interpolative reasoning, in order to interpolate **Explosion Likelihood**, the observed value for the level of **Crowdedness** and **Warning** level are both required. The antecedent variable **Warning** level is particularly important for it is required by both rule bases. Without it, no matter what other information is there available, even with the **Crowdedness** known, as illustrated in Table VII, forward interpolation would still fail.

	Crowdedness	**Warning**	**Explosion Likelihood**
E1	(0.0,1.0,2.0)	(0.0,0.8,1.8)	(0.0,0.9,1.9)
E2	(0.0,1.0,2.0)	(3.4,4.4,5.4)	(0.0,0.7,1.7)
E3	(0.1,1.1,2.1)	(7.4,8.4,9.4)	(0.0,0.6,1.6)
E4	(3.5,4.5,5.5)	(0.0,0.5,1.5)	(3.2,4.2,5.2)
E5	(3.7,4.7,5.7)	(3.1,4.1,5.1)	(2.3,3.3,4.3)
E6	(4.2,5.2,6.2)	(6.9,7.9,8.9)	(1.9,2.9,3.9)
E7	(7.6,8.6,9.6)	(0.0,0.3,1.3)	(7.2,8.2,9.2)
E8	(7.0,8.0,9.0)	(3.5,4.5,5.5)	(4.5,5.5,6.5)
E9	(7.7,8.7,9.7)	(7.6,8.6,9.6)	(3.7,4.7,5.7)

TABLE V

FUZZY RULE BASE FOR EXPLOSION LIKELIHOOD

	Popularity	**Travel**	**Warning**	**Crowdedness**
C1	(0.0,0.4,1.4)	(0.0,0.5,1.5)	(0.0,0.9,1.9)	(0.0,0.8,1.8)
C2	(0.1,1.1,2.1)	(0.0,0.6,1.6)	(3.1,4.1,5.1)	(0.0,0.8,1.8)
C3	(0.2,1.2,2.2)	(0.0,1.0,2.0)	(7.3,8.3,9.3)	(0.0,0.7,1.7)
C4	(0.0,0.4,1.4)	(3.0,4.0,5.0)	(0.0,0.9,1.9)	(0.8,1.8,2.8)
C5	(0.0,0.6,1.6)	(3.6,4.6,5.6)	(3.2,4.2,5.2)	(0.5,1.5,2.5)
C6	(0.0,1.0,2.0)	(3.8,4.8,5.8)	(7.1,8.1,9.1)	(0.4,1.4,2.4)
C7	(0.0,0.7,1.7)	(7.6,8.6,9.6)	(0.0,1.0,2.0)	(1.8,2.8,3.8)
C8	(0.1,1.1,2.1)	(7.1,8.1,9.1)	(3.7,4.7,5.7)	(1.3,2.3,3.3)
C9	(0.0,0.8,1.8)	(7.1,8.1,9.1)	(6.9,7.9,8.9)	(0.7,1.7,2.7)
C10	(3.2,4.2,5.2)	(0.0,1.0,2.0)	(0.0,1.0,2.0)	(1.1,2.1,3.1)
C11	(3.4,4.4,5.4)	(0.0,0.8,1.8)	(3.9,4.9,5.9)	(0.5,1.5,2.5)
C12	(3.5,4.5,5.5)	(0.0,0.9,1.9)	(7.3,8.3,9.3)	(0.3,1.3,2.3)
C13	(3.2,4.2,5.2)	(3.7,4.7,5.7)	(0.0,1.0,2.0)	(3.2,4.2,5.2)
C14	(3.4,4.4,5.4)	(3.2,4.2,5.2)	(3.9,4.9,5.9)	(2.0,3.0,4.0)
C15	(3.7,4.7,5.7)	(4.0,5.0,6.0)	(7.2,8.2,9.2)	(1.7,2.7,3.7)
C16	(3.1,4.1,5.1)	(7.4,8.4,9.4)	(0.0,0.9,1.9)	(4.5,5.5,6.5)
C17	(3.2,4.2,5.2)	(7.5,8.5,9.5)	(3.9,4.9,5.9)	(3.1,4.1,5.1)
C18	(3.2,4.2,5.2)	(7.7,8.7,9.7)	(6.8,7.8,8.8)	(2.5,3.5,4.5)
C19	(7.6,8.6,9.6)	(0.2,1.2,2.2)	(0.0,0.5,1.5)	(2.3,3.3,4.3)
C20	(7.4,8.4,9.4)	(0.1,1.1,2.1)	(3.0,4.0,5.0)	(1.4,2.4,3.4)
C21	(6.8,7.8,8.8)	(0.0,0.5,1.5)	(7.4,8.4,9.4)	(0.5,1.5,2.5)
C22	(7.1,8.1,9.1)	(3.9,4.9,5.9)	(0.0,0.5,1.5)	(5.0,6.0,7.0)
C23	(7.2,8.2,9.2)	(3.4,4.4,5.4)	(3.6,4.6,5.6)	(3.2,4.2,5.2)
C24	(7.4,8.4,9.4)	(3.3,4.3,5.3)	(6.8,7.8,8.8)	(2.5,3.5,4.5)
C25	(7.3,8.3,9.3)	(7.2,8.2,9.2)	(0.0,0.6,1.6)	(6.7,7.7,8.7)
C26	(7.4,8.4,9.4)	(6.8,7.8,8.8)	(3.4,4.4,5.4)	(4.7,5.7,6.7)
C27	(7.6,8.6,9.6)	(7.0,8.0,9.0)	(7.1,8.1,9.1)	(3.6,4.6,5.6)

TABLE VI

FUZZY RULE BASE FOR CROWDEDNESS

Popularity	**Travel**	**Warning**	**Crowdedness**
(6.3,6.8,7.4)	(5.6,7.4,8.1)	N/A	(4.3, 5.3, 6.2)
Moderate High	Moderate High	N/A	Moderate

TABLE VII

OBSERVATION

In order to interpolate **Explosion Likelihood**, it is essential to determine the value of **Warning** using *B-FRI*. Following the steps detailed in section III-B, the two closest rules are first selected using the consequence-biased distance measure stated in Eqn. 21, as shown in Table VIII. The intermediate fuzzy term $A^{'}_{warning} = (2.6, 3.6, 4.6)$ can be derived from the two neighbouring terms $A^{i}_{warning} = (3.4, 4.4, 5.4)$ and $A^{j}_{warning} = (0.0, 1.0, 2.0)$, by using the relative displacement factor $\lambda_{warning}$ calculated from Eqn. 23. It is then scaled and moved using $s_{warning}$, $m_{warning}$ from Eqn. 25 and 26, producing the backward interpolated value $A^{*}_{warning} = (2.7, 3.5, 4.6)$. Now with both antecedents $A^{*}_{crowdedness} = (4.3, 5.3, 6.2)$ and $A^{*}_{warning} = (2.7, 3.5, 4.6)$ present, the second rule base can therefore be evoked to produce the final interpolation result, $B^{*}_{explosion} = (3.1, 4.0, 5.0)$, this time using the standard *T-FIR*.

If the normal distance measure in Eqn. 11 is used, the rules will no longer be selected with biased focus on the consequence, instead treating everything with equal weight. The rules shown in Table IX will then be selected. Following

Popularity	Travel	Warning	Crowdedness
(7.4,8.4,9.4)	(6.8,7.8,8.8)	(3.4,4.4,5.4)	(4.7,5.7,6.7)
(3.2,4.2,5.2)	(3.7,4.7,5.7)	(0.0,1.0,2.0)	(3.2,4.2,5.2)

TABLE VIII

TWO CLOSEST RULES USING BIASED DISTANCE MEASURE

Popularity	Travel	Warning	Crowdedness
(7.1,8.1,9.1)	(3.9,4.9,5.9)	(0.0,0.5,1.5)	(5.0,6.0,7.0)
(3.2,4.2,5.2)	(7.5,8.5,9.5)	(3.9,4.9,5.9)	(3.1,4.1,5.1)

TABLE IX

TWO CLOSEST RULES USING PLAIN DISTANCE MEASURE

the backward interpolation process again, a different outcome of $A^*_{warning} = (1.0, 1.5, 2.5)$ will be derived, followed by $B^*_{explosion} = (3.6, 4.5, 5.5)$ interpolated using *T-FIR* on the second rule base. Looking back at the original observation, given the two known antecedent values $A^*_{popularity} = (6.3, 6.8, 7.4)$ and $A^*_{travel} = (5.6, 7.4, 8.1)$, the intuitive deduction of **Crowdedness** should be quite high, as the place is both moderately high in popularity and is moderately convenient to reach. The only reason that the observed **Crowdedness** is having a moderate value of $(4.3, 5.3, 6.2)$ may well have been caused by a reasonable level of **Warning**. Intuitively the outcome $A^*_{warning} = (2.7, 3.5, 4.6)$ from the biased distance measure is therefore more agreeable than $A^*_{warning} = (1.0, 1.5, 2.5)$ from the plain distance measure. Further experiments show that, by using $A^*_{warning} = (1.0, 1.5, 2.5)$ and the two known antecedent values, *T-FIR* method interpolates **Crowdedness** as $(5.39, 6.34, 7.18)$, which is much further than the original observation.

V. CONCLUSION

This paper has presented a backward fuzzy rule interpolative reasoning approach implemented using the scale and move transformation-based fuzzy interpolation method. It offers a means to broaden the application of fuzzy rule interpolation and fuzzy inference. The proposed technique allows flexible interpolation when certain antecedents are missing from the observation, where traditional approach fails. Two numerical examples are provided to illustrate the operation of this approach. A practical application is also included to demonstrate the feasibility of the proposed approach in potentially addressing real-world problems.

Currently, the work is only applicable when using triangular membership functions and two adjacent rules. It does not cover the issue of extrapolation either. Extensions which enables this approach to cover these important *FRI* aspects remain as active research. Further more, it would be useful to have a generalised approach that can be implemented using other type of interpolation method (e.g. IMUL [15], FIVE [11], or GM [1]), with results compared. The proposed method may also be further generalised [13] and combined with the adaptive fuzzy interpolation technique which ensures inference consistency [16]. Finally, it is very interesting to investigate how the proposed work may be used to form a theoretical basis upon which a hierarchical fuzzy interpolation mechanism could be developed, which would allow more effective and efficient use of rule bases involving fuzzy rules of different length or variables.

REFERENCES

[1] P. Baranyi, L. T. Kóczy, and T. D. Gedeon, "A Generalized Concept for Fuzzy Rule Interpolation," *IEEE Trans. on Fuzzy Systems*, pp. 820-837, 2004.

[2] P. Baranyi, D. Tikk, Y. Yam, and L. T. Kóczy, "A new method for avoiding abnormal conclusion for α-cut based rule interpolation," in *FUZZ-IEEE'99, 1999*, 1999, pp. 383-388.

[3] B. Bouchon-Meunier, J. Delechamp, C. Marsala, Radko Mesiar and M. Rifqi, " Fuzzy deductive reasoning and analogical scheme," in *EUSFLAT-ESTYLF Joint Conf.*, 1999, pp. 139-142,.

[4] S. M. Chen and Y. C. Chang, "A new method for fuzzy rule interpolation based on the ratio of fuzziness of interval type-2 fuzzy sets," in *Fuzzy Systems (FUZZ), 2010 IEEE International Conference*, 2010, pp. 1-6.

[5] Z. Huang and Q. Shen, "Fuzzy interpolative reasoning via scale and move transformations," *IEEE Trans. Fuzzy Syst*, pp. 340–359, 2006.

[6] Z. Huang and Q. Shen, "Fuzzy interpolation and extrapolation: a practical approach," *IEEE Trans. Fuzzy Syst*, pp. 13–28, 2008.

[7] Z. C. Johanyák, Sz. Kovács, "A brief survey and comparison various interpolation based fuzzy reasoning methods," *Acta Politechnica Hungarica, Journal of Applied Sciences at Budapest Tech Hungary*, pp. 91-105, 2006.

[8] E. P. Klement, R. Mesiar and E. Pap, "Integration with Respect to Decomposable Measures, Based on a Conditionally Distributive Semiring on the Unit Interval," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, pp. 701-717, 2000.

[9] L. T. Kóczy and K. Hirota, "Approximate reasoning by linear rule interpolation and general approximation," *Int. J. Approx. Reason*, pp. 197–225, 1993.

[10] L. T. Kóczy and K. Hirota, "Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases," *Inform. Sci.*, pp. 169–201, 1993.

[11] S. Kovács, L. T. Kóczy, "Application of an approximate fuzzy logic controller in an AGV steering system, path tracking and collision avoidance strategy," *Fuzzy Set Theory and Applications, In Tatra Mountains Mathematical Publications, Mathematical Institute Slovak Academy of Sciences*, pp. 456-467, 1999.

[12] S. Kovács, "Special Issue on Fuzzy Rule Interpolation," *Journal of Advanced Computational Intelligence and Intelligent Informatics*, pp. 253, 2011.

[13] Q. Shen and L. Yang, "Generalisation of scale and move transformation-based fuzzy interpolation," *Advanced Computational Intelligence and Intelligent Informatics*, pp. 288-298, 2011.

[14] D. Tikk and P. Baranyi, " Comprehensive analysis of a new fuzzy rule interpolation method," *IEEE Trans. Fuzzy Syst*, pp. 281-296, 2011.

[15] K. W. Wong, D. Tikk, T. D. Gedeon and L. T. Kóczy, "Fuzzy rule interpolation for multidimensional input spaces with applications: A case study," *IEEE Trans of Fuzzy Systems*, pp. 809-819, 2005.

[16] L. Yang and Q. Shen, "Adaptive Fuzzy Interpolation," *To appear in IEEE Transactions on Fuzzy Systems*, 2011.

[17] L. A. Zadeh, "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Trans. Syst., Man, Cybern*, pp. 28-44, 1973.